Physics 20

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Module 7
OSCILLATORY MOTION

Rocky View School Division
Red Deer Catholic Regional Schools
Edmonton Public Schools
Pembina Hills Regional School Division No. 7
Black Gold Regional Schools
Peace Wapiti School Division No. 76
Calgary Board of Education
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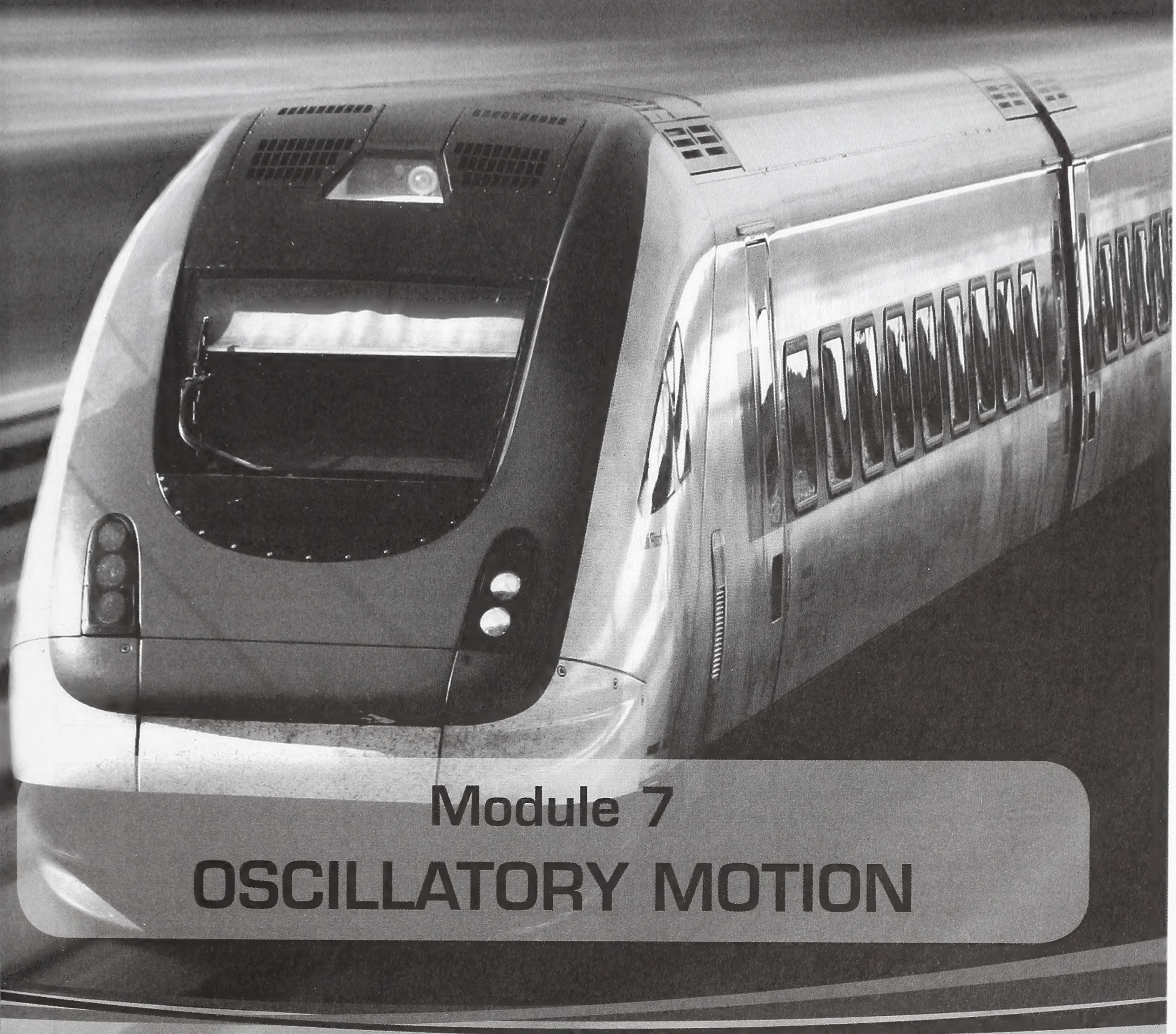

Module 7
OSCILLATORY MOTION

Red Deer Catholic Regional Schools

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- Black Gold Regional Schools
- Calgary Board of Education
- Edmonton School District No. 7
- Peace Wapiti School Division No. 76
- Pembina Hills Regional Division No. 7
- Red Deer Catholic Regional Division
- Rocky View School Division No. 41

Physics 20
Module 7: Oscillatory Motion
Student Module Booklet
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This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

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Contents

Physics 20

Unit D Oscillatory Motion and Mechanical Waves

Unit D Introduction

This unit is about 25% of the Physics 20 course. This is equivalent to approximately 20 80-minute classes. You may have to do additional "homework" hours to complete this unit.

In this unit you will investigate simple harmonic motion and mechanical waves. This will prepare you for further study of simple harmonic motion and wave phenomena in other physics courses.

The major concepts developed in this unit are for you to be able to
- describe the conditions that produce oscillatory motion
- describe the properties of mechanical waves and explain how they transmit energy

Think about the following questions as you complete this unit:
- What are examples of oscillatory motion in the world around you?
- How do mechanical waves transmit energy?
- How is structural design and development of technologies influenced by the understanding of wave properties?

Concept Organizer

Unit D – Oscillatory Motion and Mechanical Waves

Module 7: Oscillatory Motion
Lesson 1 – Simple Harmonic Motion of a Weighted Spring
Lesson 2 – Simple Harmonic Motion of a Pendulum
Lesson 3 – Mechanical Resonance

Module 8: Mechanical Waves
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Module Descriptions

Module 7: Oscillatory Motion

In this module you will examine the conditions that produce oscillatory motion and begin your study of simple harmonic motion.

The essential question that you will be looking at in this module is the following:

- What are examples of oscillatory motion in the world around you?

Module 8: Mechanical Waves

In this module you will study properties of mechanical waves. You will learn how mechanical waves transmit energy and how this can help provide solutions to practical problems in your life.

The essential questions that you will be looking at in this module are the following:

- How do mechanical waves transmit energy?

- How is structural design and development of technologies influenced by the understanding of wave properties

Module 7—Oscillatory Motion

Module Introduction

In this module you will examine the conditions that produce oscillatory motion and begin your study of simple harmonic motion.

The essential question that you will be looking at in this module is the following:

- What are examples of oscillatory motion in the world around you?

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Big Picture

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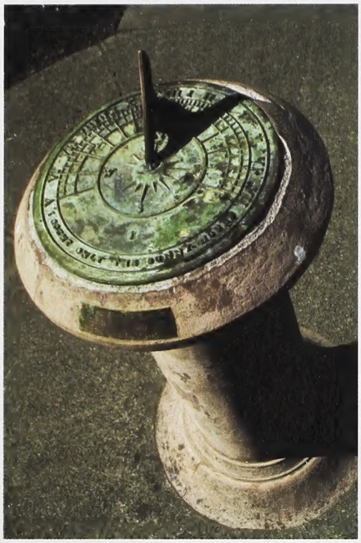
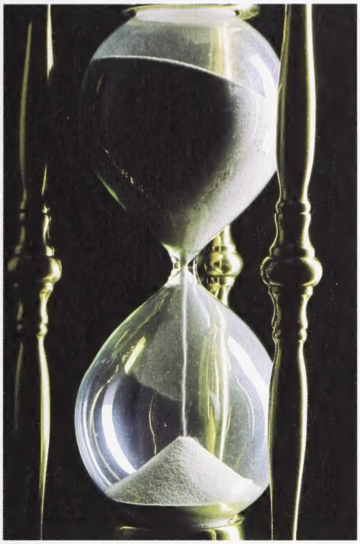
Passengers are in an airport terminal.

In today's technologically oriented world, knowing the time is essential. For example, work starts at 8:00, lunch is at 12:00, and the news is on at 6:00. The frozen pizza stays in the oven 24 minutes. The boarding gate opens at 9:30, and the plane to Madrid leaves at 10:05. In the past, being aware of time in such a way would have been impossible. In many places around the world, it would not be considered wise to let time control you instead of the other way around.

Technology has made this awareness to time both possible and inevitable. Without accurate ways to measure and show time, how could we coordinate starting work at 8:00? Having lunch at noon works because the Sun is highest in the sky then, but would people in Edson be eating lunch later than those in Chauvin?

Just how do we keep time accurately enough for the world to function as it does? Basically, it's as simple as finding something that repeats consistently and regularly. However, it has taken centuries for timekeeping to evolve to its current level of sophistication.





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The repeating pattern was clear in each device that was used, and the length of time for one repetition was usually smaller as each new device was created. The sundial took a day to complete one repetition; the hourglass took an hour; and, depending on when they were constructed, large community clocks took an hour or less.

Once, time-keeping devices were not commonplace. The grandfather clocks shown here used a pendulum as the source of the regular, repeating pattern. However, the pendulum had its drawbacks. These clocks were so large that you couldn't take them with you when you went out; and even when smaller clocks were developed, they would not keep time unless they were stationary. The motion of carrying the clock interfered with the motion of the pendulum. Eventually the pendulum was replaced with new technologies. Metallurgy and tool making also advanced to the point that small timepieces that are easily worn were available.

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Those evenly spaced, repetitive events that gave us time and clocks are useful in other ways too. Have you ever taken music lessons?

If so, you will have learned how important the tempo of the music you are playing is. Time is an important part of western music. The length of each note is an integral part of the song. If you change the lengths of some notes and not others, it just doesn't sound right. The simple metronome that creates a steady beat can help you play music just the way the composer intended it to be played.

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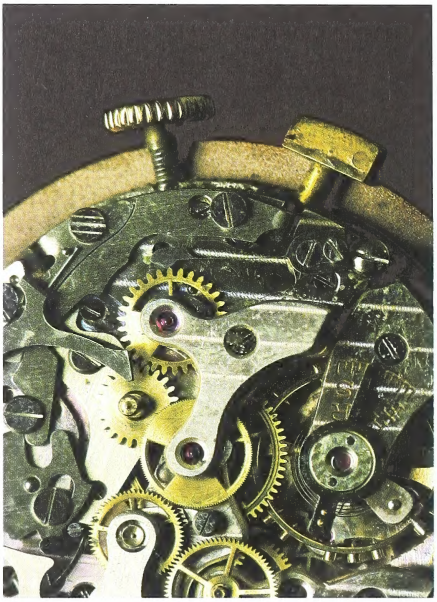

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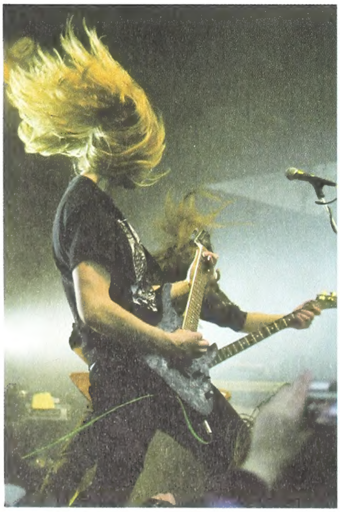
Is that the only place that steady, repetitive events come into music? Actually, no. The very sound that you are making is made up of waves that repeat similar patterns hundreds of times for each note you play.

All of the musicians in these pictures use repetition—to get to the concert on time, to define the tempo of their music, and to create vibrations that initiate sound waves—in making music that their fans enjoy.



left; © Andreas Gradin/shutterstock

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As you work through this module, keep the following questions in mind. They should help you better understand oscillatory motion in your world.

- What is oscillatory motion?
- What is simple harmonic motion?
- What is the equation for the period of a weighted spring?
- How is simple harmonic motion related to circular motion?
- Is the motion of a pendulum simple harmonic motion?
- What is the equation for the period of a pendulum?
- How is the restoring force of a pendulum determined?
- How is simple harmonic motion related to circular motion?
- What is a resonant frequency?
- What is mechanical resonance?
- Are there examples of mechanical resonance in your everyday world?

In This Module

Lesson 1—Simple Harmonic Motion of a Weighted Spring

In this lesson you will begin your study of oscillatory motion. Specifically, you will examine the simple harmonic motion exhibited by a mass-spring system. You will use a simulation that will help you understand simple harmonic motion, and you will apply Hooke's law to help you solve related problems.

- What is oscillatory motion?
- What is simple harmonic motion?
- What is the equation for the period of a weighted spring?
- How is simple harmonic motion related to circular motion?

Lesson 2—Simple Harmonic Motion of a Pendulum

In this lesson you will expand your understanding of simple harmonic motion to include pendulums. You will complete a lab that will help you to determine the equation for the period of a pendulum. Finally, you will examine the relationship between simple harmonic motion and circular motion.

- Is the motion of a pendulum simple harmonic motion?
- What is the equation for the period of a pendulum?
- How is the restoring force of a pendulum determined?
- How is simple harmonic motion related to circular motion?

Lesson 3—Mechanical Resonance

In this lesson you will be examining the positive and negative effects of mechanical resonance.

- What is resonant frequency?
- What is mechanical resonance?
- Are there examples of mechanical resonance in your everyday world?

Module 7 Assessment

The assessment for Module 7 consists of three (3) assignments, as well as a module project

- Module 7: Lesson 1 Assignment
- Module 7: Lesson 2 Assignment
- Module 7: Lesson 3 Assignment
- Module 7 Project

Module 7 Project

As you work through this module, each lesson has a Reflect on the Big Picture section with a choice of activities supporting reflection and consolidation of your learning about oscillation. For your module project, you will choose one of the Big Picture Reflection activities that best represents your understanding of oscillation, and then edit and submit it to your teacher for marks. When you have completed the module, continue on to the Module Assessment section of the Module Summary for more details.

Lesson 1—Simple Harmonic Motion of a Weighted Spring

Get Focused


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The guitar in this photo has several oscillating, or vibrating, strings. Each string will vibrate with a specific frequency once disturbed by the person playing it. When you hear different strings vibrating with different frequencies, you hear distinctive sounds. There are three ways to make different pitches on a guitar—you can vibrate different strings; you can change the length of the strings by pressing your fingers down on the frets (thin metal strips embedded on the fret board, seen in the foreground of the photo); or you can change the sound by "tuning" or adjusting the tuning screws at the end of the guitar neck. The third method changes the tension in the strings, causing them to vibrate with a different frequency.

Merely causing strings to vibrate does not always produce a pleasant sound. However, vibrating just the right strings, that are at just the right length, with just the right tension, can be music to your ears. All three methods for producing various sounds are related by one simple concept—simple harmonic motion.

In this lesson and the accompanying lab, you will investigate the following questions:

- What is oscillatory motion?
- What is simple harmonic motion?
- What is the equation for the period of a weighted spring?
- How is simple harmonic motion related to circular motion?

Module 7: Lesson 1 Assignments

In this lesson you will complete the Lesson 1 Assignment in the Module 7 Assignment Booklet.

- Try This—TR 1, TR 2, TR 3, TR 4, and TR 5
- Lab—LAB 1, LAB 5, LAB 6, LAB 7, LAB 8, and LAB 9

The other questions in this lesson are not marked by the teacher; however, you should still answer these questions. The Self-Check and Try This questions are placed in this lesson to help you review important information and build key concepts that may be applied in future lessons.

After a discussion with your teacher, you must decide what to do with the questions that are not part of your assignment. For example, you may decide to submit to your teacher the responses to Try This questions that are not marked. You should record the answers to all the questions in this lesson and place those answers in your course folder.

Explore

Oscillatory Motion

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What does a flying insect have in common with a guitar? Not much, you would think, but they both make sound. What about flying birds, idling car engines, electrical transformer boxes, fridges, freezers, electric shavers, and lawnmowers? They, too, produce steady sounds when they are in operation. This is because they **oscillate** with a constant **frequency** (f). An oscillation is one complete cycle of a motion or vibration. For example, the wings of an insect move up and down, completing one cycle every time they reach their original starting position. The cycle of a guitar string occurs in much the same way—the string moves up, then down, and then returns to its original starting position. Recall from Module 5: Lesson 1 that the number of cycles that occur in a certain amount of time is called the frequency (f). If the time period is one second, the frequency is measured in hertz (Hz).

frequency = cycles per time period
1 Hz = 1 cycle/second
1 Hz = 1/s

oscillate: to move back and forth at a constant rate

The time required to complete one cycle, which is known as the **period** (T), is the reciprocal of the frequency. Frequency and period are related mathematically by the following equation:

$$T = \frac{1}{f}$$

Consider an electrical transformer that vibrates with a constant frequency of 60 Hz. What is the cause of this vibration? The alternating current flowing in the windings (the electrical wires) of the transformer causes the vibration. In Alberta, that current makes the change from positive to negative and back to positive, one complete cycle, 60 times every second; therefore, the frequency of vibration is 60 Hz. The period of vibration is 1/60 Hz = 0.0167 s. This means the electrical current reverses direction twice every 0.0167 s.

Read

Read pages 344 to 347 in your textbook to review how frequency and period are converted from one to another.

Self-Check

SC 1. Complete question 2 of "Practice Problems" on page 345 of your textbook.

SC 2. Complete question 9 of "7.1 Check and Reflect" on page 347 of your textbook.

SC 3. Complete "7-3 QuickLab: Determining the Stiffness of a Spring" beginning on page 348 of your textbook with another student, friend, or family member. Ask your teacher for the materials. Then complete questions 1, 2, 3, 4, and 5 on page 349.

Check your work with the answer in the appendix.

Simple Harmonic Motion and Hooke's Law

A guitar will not produce music unless someone is playing it. So what does it mean to "play" a guitar? Essentially, the player has to apply a series of forces to the strings, "plucking" them with a pick or finger. This force disturbs the string, causing it to deform or stretch from the **equilibrium position**. When this is done, the spring stores energy, and a **restoring force** acts to return the string to its equilibrium position. Whenever a restoring force is present, the oscillatory motion is called **simple harmonic motion**. For example, there is a distinct difference between the oscillatory motion of an insect's wings and the simple harmonic motion of a vibrating spring, such as a guitar string. The insect's wings do not have a restoring force that is proportional to the distance they are moved from equilibrium. In other words, they are not springs.

equilibrium position: the position where the resultant of all forces acting is zero

restoring force: a force that causes an object to return to an equilibrium position

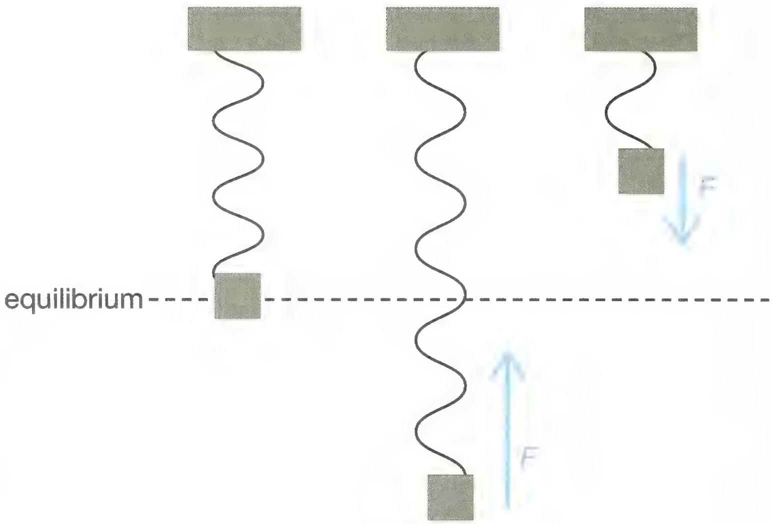
simple harmonic motion: a repeating motion about a central equilibrium point caused by restoring forces

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Harmonic Motion and Music." This video introduces the relationship between simple harmonic motion and music.

Read

Consider the weighted spring that hangs at the equilibrium position, as seen in the Watch and Listen video.



The spring is given potential energy when the weight is pushed up or pulled down. If the weight is pulled down, the spring pulls the weight back toward the equilibrium position when it is released.

The potential energy is converted into kinetic energy until the weight reaches its maximum speed at the equilibrium position. The weight then continues through the equilibrium position to end up at rest for an instant on the opposite side from where it started. The kinetic energy has been converted back into potential energy. Then, gravity pulls the weight back to the equilibrium position, with gravity continuing to pull the weight until it reaches its starting position again. One cycle or vibration has been completed.

A weighted spring exhibits simple harmonic motion. This means the spring is vibrating with a constant frequency or period of motion and there is a restoring force directed towards the central equilibrium point. Furthermore, the magnitude of the restoring force is proportional to the displacement from the equilibrium point.

One other factor that must be considered in your study of simple harmonic motion relates to the spring itself. You have no doubt stretched different thicknesses of elastics. Some stretch very easily, and others require much more force to get them to stretch. The same is true of springs. This is referred to as the **spring constant**. The spring constant tells you how stretchy (or elastic) a material is. It is the "stiffness" of a spring and can also be called the **force constant**.

The following simulation will be used to investigate the direction and magnitude of the restoring force. The applet used in this simulation helps you explore the motion of a weighted vertical spring.

spring constant: a measure of the stiffness or strength of a spring (the k in Hooke's law)

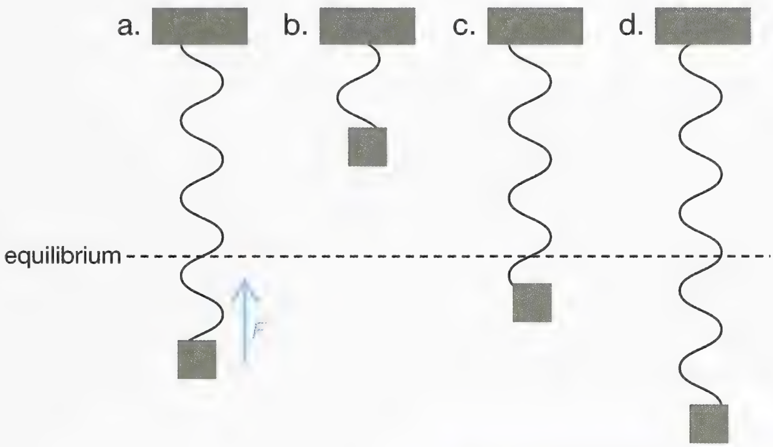
Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher fo this information. Enter the search terms "Simple Harmonic Motion Weighted Spring" in the search bar. Choose the item called "Simple Harmonic Motion: Weighted Spring (Grade 11)." Open the simulation and continue with TR 1. You can learn more about the simulation and how to use it by reading the Show Me found at the top of the simulation screen.

Module 7: Lesson 1 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 1 Assignment in your Module 7 Assignment Booklet.

Try This

TR 1. To set up the simulation, click on the "Vectors" button (Vectors) and choose "acceleration at origin" on the popup menu. If the "Selected Vectors" popup menu does not display "acceleration at origin", drag the green bar at the top of the popup upwards till all the choices are visible. Then click on the "Components" button (Components) to show the acceleration vector. Now press "Play," and observe the motion of the weighted spring and the corresponding acceleration vector. The acceleration vector is proportional in magnitude to the restoring force by Newton's second law ($F = ma$). Based on your observations, draw the restoring force on each of the images below. The first one has been completed as an example.



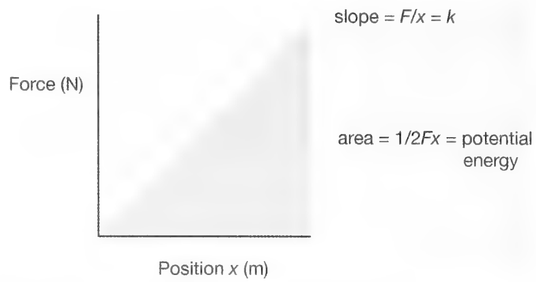
For a weighted spring, the magnitude of the restoring force is determined by **Hooke's law**. Expressed as an equation, it is

Hooke's law: the amount of deformation (compression or extension) of an elastic object is proportional to the force applied to deform it

$$\vec{F} = -k\vec{x}$$

Quantity	Symbol	SI Unit
force	F	N
spring constant	k	N/m
displacement (amount of deformation)	x	m

Graphing the force required to compress a spring vs. the displacement gives a slope that is equal to the spring constant.



Because the restoring force always acts in the opposite direction to the displacement, when using Hooke's law to represent the restoring force, you must use $-x$ in the formula $\vec{F} = -k\vec{x}$.

The slope of the line that you determined in "7-3 QuickLab" for SC 3 is the spring constant (k) for the spring you used.

Example Problem 1

A spring with a force constant of 55.0 N/m has a 0.250-kg mass suspended from it. How far does the spring stretch?

Solution

$$
\begin{aligned}
F &= -kx \\
x &= -\frac{F}{k} \\
x &= -\frac{mg}{k} \\
x &= -\frac{(0.250\ \text{kg})(9.81\ \text{m/s}^2)}{55.0\ \text{N/m}} \\
x &\doteq -0.044\ 590\ 909\ 1 \\
x &= -0.0446\ \text{m}\,(\text{correct to 3 signficant digits})
\end{aligned}
$$

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Harmonic Motion and Music 2."

Read

Read "Simple Harmonic Motion" and "Hooke's Law" on pages 348 to 353 of your textbook.

Self-Check

SC 4. Complete question 2 of "Practice Problems" on pages 352 of the textbook.

Check your work with the answer in the appendix.

Read

Why is the spring constant important? Read "The Restoring Force" on pages 353 and 354 of the textbook.

Self-Check

SC 5. Complete question 2 of "Practice Problems" on pages 354 of the textbook.

Check your work with the answer in the appendix.

Read

In TR 1 you looked at the motion of a spring in the vertical position, where a restoring force kept bringing the object back to the equilibrium position. You saw that the farther the object was from the equilibrium position, the greater the restoring force and the acceleration of the object.

How is it different when a spring is in the horizontal plane than when it is in the vertical plane? Read "Simple Harmonic Motion of Horizontal Mass-spring Systems" on pages 354 and 355 in your textbook.

Self-Check

SC 6.

a. What forces are involved in the horizontal case?
b. How is that different than the vertical case?

Check your work with the answer in the appendix.

Read

Read "Simple Harmonic Motion of a Vertical Mass-Spring Systems" on pages 356 to 359 of your textbook.

Self-Check

SC 7. Complete question 1 of "Practice Problems" on page 357 of the textbook. In this vertical case, the springs are compressed because the weight is above them instead of being stretched when a weight is suspended from the spring as in the previous example.

Check your work with the answer in the appendix.

The Period of a Weighted Spring

What makes some springs bob fast and others more slowly? Find out what influences the time it takes for a spring to oscillate by completing the following lab.

Lesson 1 Lab: Equation for the Period of a Weighted Spring

Problem

What is the equation for the period of a weighted spring?

The applet used in this simulation helps you explore the motion of a weighted vertical spring.

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Simple Harmonic Motion Weighted Spring" in the search bar. Choose the item called "Simple Harmonic Motion: Weighted Spring (Grade 11)." Reopen the simulation, if necessary. You can learn more about the simulation and how to use it by reading Show Me found at the top of the simulation screen. Continue with the Procedure.

Procedure and Observations

Using the simulation, determine the period of the weighted spring by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Press "Play," and carefully watch the spring move through ten complete cycles. (One cycle is complete every time the weight passes the starting position on its way upward.)
- Stop the applet at the end of the tenth cycle.

Module 7: Lesson 1 Assignment

Remember to submit the answer to LAB 1 to your teacher as part of your Lesson 1 Assignment in your Module 7 Assignment Booklet.

LAB 1. In the Period Measurements table, record the time for ten cycles in the simulation. The time required to complete one cycle is the period of the weighted spring. Calculate the period from the data for ten cycles. Record the data under the column heading "With Default Settings." You should save the Period Measurements table to your course folder. You will update the table in LAB 2, LAB 3, LAB 4, and LAB 5.

Number of Cycles	**Time to Complete (Seconds)**			
	With Default Settings	With Modified **Amplitude of Release**	With Modified **Mass**	With Modified **Spring Constant**
10				
1				

Now that you know this, you will systematically investigate the effects of changing the amplitude of release (x), the mass (kg), and the spring constant (k).

Using the simulation, determine if the period of the weighted spring is affected by the amplitude of release by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Change the amplitude of release to the maximum value of 0.30 m ( ).
- Press "Play," and observe ten complete cycles. (One cycle is complete every time the weight passes the starting position on its way up.)

LAB 2. In the table, record the time for ten cycles from the data display. Record the data under the heading "With Modified Amplitude of Release."

On the simulation, determine if the period of the weighted spring is affected by the mass by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Change the mass of the weight to the maximum value of 2.00 kg ( ).
- Press "Play," and observe ten complete cycles. (One cycle is complete every time the weight passes the starting position on its way up.)

LAB 3. In the table, record the time for ten cycles from the data display. Record the data under the heading "With Modified Mass."

On the simulation, determine if the period of the weighted spring is affected by the spring constant by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Change the spring constant to the maximum value of 200 N/m ( ).
- Press “Play,” and observe ten complete cycles.

LAB 4. In the table, record the time for ten cycles from the data display. Record the data under the heading “With Modified Spring Constant.”

Module 7: Lesson 1 Assignment

Remember to submit the answer to LAB 5 to your teacher as part of your Lesson 1 Assignment in your Module 7 Assignment Booklet.

LAB 5. Find the average time for the completion of one cycle for each of the previous steps of the procedure. (You do this by dividing the time for ten cycles by 10.) Place your results in the appropriate cells in the Period Measurements table. You will submit your completed table to your teacher for marks.

Module 7: Lesson 1 Assignment

Remember to submit the answers to LAB 6, LAB 7, LAB 8, and LAB 9 to your teacher as part of your Lesson 1 Assignment in your Module 7 Assignment Booklet.

Analysis

LAB 6. Has the period changed as a result of changing the amplitude of release? Explain.

LAB 7. Has the period changed as a result of changing the mass? Explain.

LAB 8. Has the period changed as a result of changing the spring constant? Explain.

LAB 9. Summarize your findings from LAB 6, LAB 7, and LAB 8 by listing the parameters that *do* affect the period of the weighted spring and the ones that *do not* have an effect.

Conclusion

The period of a weighted spring is the time required to complete one cycle. It is defined by the spring constant and the mass of the weight attached to the spring. Expressed as an equation, it is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Quantity	Symbol	SI Unit
period (The period depends on the stiffness of the spring (spring constant) and the mass of the hanging object.)	T	s
mass of the weight	m	kg
spring constant	k	N/m

This equation tells you that a larger mass will oscillate more slowly (large period) and a stiff spring will oscillate more quickly (small period).

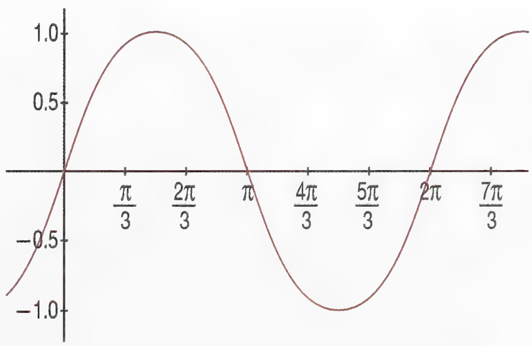
Did you wonder about the 2π in the equation? Where does it come from? It looks like something involving a circle, doesn't it? How is that related to the motion of a mass suspended from a spring?

Simple Harmonic Motion Is Sinusoidal

Is simple harmonic motion an application of circular motion? If so, how are they related? Examining the velocity, acceleration, and position of a weighted spring as it oscillates will reveal any relationships. Tracking the velocity of the weighted spring as it vibrates through several cycles produces a curve that has precisely the same shape as a sinusoidal curve, so-called because it is like the graph of the sine function in trigonometry. That is, simple harmonic motion is sinusoidal in nature.

Simple harmonic motion is also based on uniform circular motion. Note that the *x*-axis on the illustration shows **radian** units. Why is this? The circumference of a circle is equal to 2π radians; and for each cycle of the weighted spring, it completes one circle.

radian: a unit used to measure angles that is calculated as arc length divided by radius



Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Graphing Cyclic Data" in the search bar. Choose the item called "Graphing Cyclic Data (Grade 12)." If you would like to know more about radian measure than just $2\pi = 360°$ $\left(\text{or equivalently } \pi = 180°,\ \frac{\pi}{2} = 90°,\ \frac{\pi}{3} = 60°,\ \frac{\pi}{4} = 45°\right)$, click "Start" beside Graphing Cyclic Data.

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Simple Harmonic Motion Weighted Spring" in the search bar. Choose the item called "Simple Harmonic Motion: Weighted Spring (Grade 11)." Reopen the simulation, if necessary, and observe uniform circular motion and simple harmonic motion at the same time by doing the following:

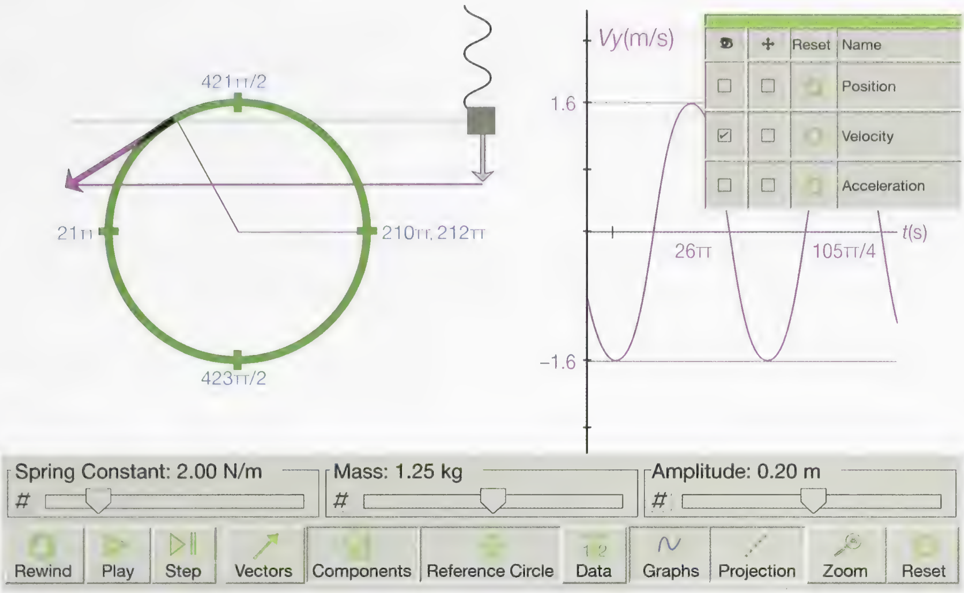
- Reset the applet ( ).
- Display the reference circle ( ), vector components ( ), and projection lines ( ).
- Press the "Vectors" button (Vectors), and select "velocity at particle" from the popup menu. You may have to drag the green bar from the popup to the upper right of the window so you can see the full popup and the full circle.
- Press "Play."

Self-Check

SC 8. Describe one similarity and one difference between the velocity vector on the reference circle and the velocity vector on the weighted spring.

Check your work with the answer in the appendix.

Without changing any of the settings in the simulation, turn on the graphing function (Graphs), and select "Velocity," as illustrated below. Then close vectors popup by clicking on the "Close" button. Close the graphing popup by pressing the "Graphs" button (Graphs).



Review the questions below so you know what to look for, and click "Play."

Module 7: Lesson 1 Assignment

Remember to submit the answer to TR 2 to your teacher as part of your Lesson 1 Assignment in your Module 7 Assignment Booklet.

Try This

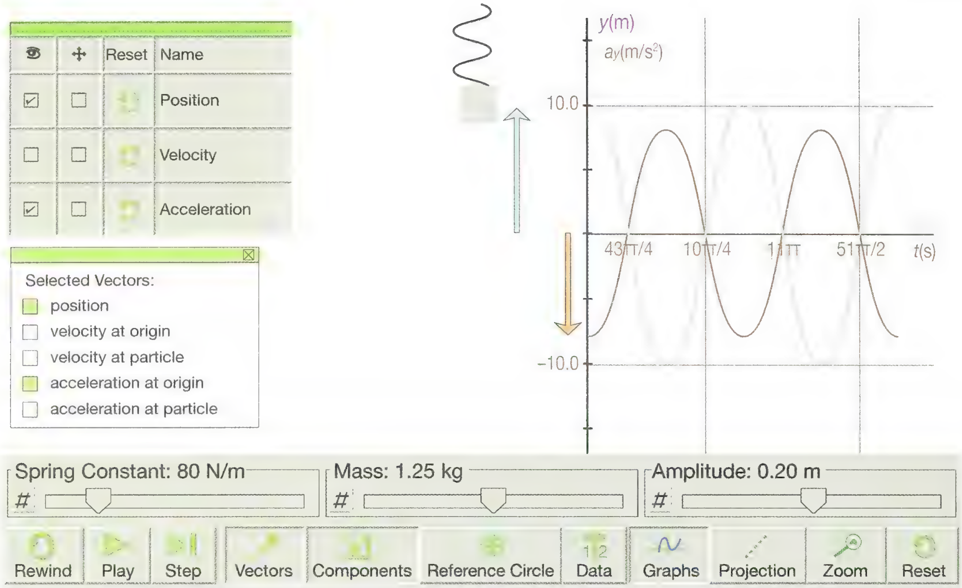
TR 2.

a. How many rotations does the reference circle make for every complete wave ( ) drawn on the graph?

b. How many complete cycles does this represent on the weighted spring? Does this mean that the period for the circular motion is identical to the period of the simple harmonic motion?

c. Compare the radius of the circle with the amplitude of the oscillator. How are they similar?

Set up the simulation as indicated in the following diagram by clicking the "Vectors" button (Vectors) popup. Choose the "position" and "acceleration at origin" selections; then click the "Vectors" button to close the popup. Next, click the graphing function (Graphs), and select the "Position" and "Acceleration" functions, as illustrated below. Then click the "Graphs" button to close the popup.



Press “Play,” and observe the orientation of the acceleration and displacement vectors.

Self-Check

SC 9. Explain how the orientation of the acceleration and displacement is related to the negative sign in $\vec{F} = -k\vec{x}$.

Check your work with the answer in the appendix.

Read

How can you calculate the maximum speed of a mass on a spring, and where does the maximum occur? Read pages 366 to 372 in your textbook to find out.

Self-Check

SC 10. Complete question 1 of “Practice Problems” on page 372 of the textbook.

SC 11. Complete question 2 of “Practice Problems” on page 372 of the textbook.

Check your work with the answer in the appendix.

Module 7: Lesson 1 Assignment

Remember to submit the answer to TR 3 to your teacher as part of your Lesson 1 Assignment in your Module 7 Assignment Booklet.

TR 3.

a. A 250-g object hangs from a spring and oscillates with an amplitude of 5.42 cm. If the spring constant is 48.0 N/m, determine the acceleration of the object when the displacement is 4.27 cm [down].

b. If the spring constant is 48.0 N/m, determine the maximum speed. Tell where the maximum speed will occur.

Read

At the start of this lesson, the frequency vibration of guitar strings was mentioned in reference to simple harmonic motion. Then, in the lab, you determined the factors that influence the period and frequency of a weighted spring. Find out how to determine the value of the frequency and period of the mass-spring system by reading pages 373 to 376 in your textbook.

Following are two other examples of how this is done.

Example Problem 2

Calculate the period of oscillation for a 6.00-kg mass hanging on a spring with a spring constant of 75.0 N/m.

Solution

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$T = 2\pi\sqrt{\frac{6.00\ \text{kg}}{75.0\ \text{N/m}}}$$
$$T \doteq 1.777\,153\,175\ \text{s}$$
$$T = 1.78\ \text{s}\ (\text{correct to 3 significant digits})$$

Example Problem 3

A 1.00-kg mass hangs from a spring and oscillates with a frequency of 10.0 Hz. Calculate the spring constant.

Solution

$$T = 2\pi\sqrt{\frac{m}{k}} \text{ and } T = \frac{1}{f}$$

$$T^2 = 4\pi^2\frac{m}{k}$$

$$k = \frac{4\pi^2 m}{T^2}$$

$$k = \frac{4\pi^2 m}{\left(\frac{1}{f}\right)^2}$$

$$k = \frac{4\pi^2(1.00 \text{ kg})}{\left(\frac{1}{10.0 \text{ Hz}}\right)^2}$$

$$k \doteq 3947.841\ 76 \text{ kg/s}^2$$

$$k = 3.95 \times 10^3 \text{ N/m (correct to 3 significant digits)}$$

Self-Check

SC 12. An object hangs from a spring and oscillates with a frequency of 3.50 Hz. If the spring constant is 24.5 N/m, what is the mass of the object?

Check your work with the answer in the appendix.

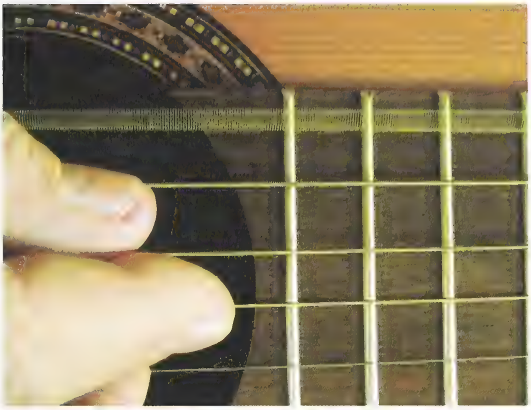
Module 7: Lesson 1 Assignment

Remember to submit the answers to TR 4 and TR 5 to your teacher as part of your Lesson 1 Assignment in your Module 7 Assignment Booklet.

TR 4. A 78.5-kg man is about to complete a bungee jump. If the bungee cord has a spring constant of 150 N/m, determine the period of oscillation that he will experience.

TR 5. A 5.00-kg mass oscillates on a spring with a frequency of 0.667 Hz. Calculate the spring constant.

Reflect and Connect

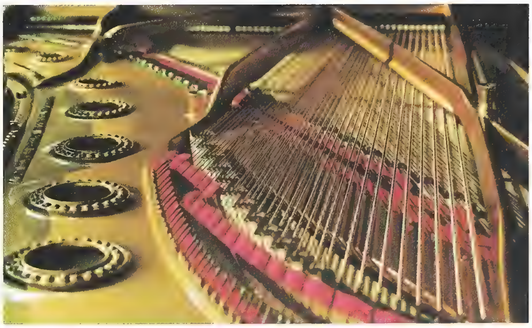

© Zoulou 55/shutterstock

A musician plucks one string on a guitar. A note is produced and heard by a nearby audience. As long as the string is left undisturbed, it will continue to exhibit simple harmonic motion—defined by a constant period and frequency. Over time, it is perfectly reasonable to expect the string vibrations to stop, but do they ever slow down? The audience observes a constant pitch emanating from the string, but the volume of the sound drops off until, moments later, it cannot be heard. Did the frequency change, or did the amplitude change?

In order to understand this, you first need to understand the difference between volume and pitch as they relate to sound. The volume is related to the amplitude of the oscillation, which decreases over time. The pitch is related to the frequency, which does not change over time. Why?

As you have discovered, the period of a simple harmonic oscillator, such as a weighted spring, is not related to amplitude. And since frequency is the inverse of the period, it is not related either. Therefore, as the amplitude of the oscillation drops off over time, the frequency remains constant. The pitch never changes, but the volume drops off as the amplitude of each oscillation is reduced until it reaches zero.

Discuss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The strings of a piano all have different masses. In the discussion forum, explain the working principles of a piano and include an explanation for all the different pitches that can be produced by a piano. Be sure to answer the following questions in your explanation.

- What difference in sound is observed when you strike the keys on the piano with different amounts of pressure?
- How can you change the pitch of any one string?
- What variable in the equation for the period of simple harmonic oscillator are you adjusting when you tune a piano string?

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will help you reinforce your learning about oscillation. Complete at least one of these reflection activities:

- Have you ever seen or driven over a washboard section of road or highway? The tires on your vehicle begin to bounce up and down as you pass over the crests and valleys of the washboard, and your tires begin to oscillate. If you are travelling too fast over a washboard section of road, you may lose control of your vehicle. How could the speed of your vehicle cause you to lose control? (**Hint:** Consider the time necessary for the restoring force to act.)

- Collect six pictures. Collect three pictures of things (naturally occurring or human made) that have a very high frequency, and collect three pictures of things that have a very low frequency. Each item oscillates at a frequency. Write the approximate frequency for each item on its picture.

- Name six instruments from the classical orchestra and six instruments that are used in popular music today. For each instrument, give the approximate frequency range that it can produce. If you had to play a popular song using only instruments from the classical orchestra, which instruments would you use? Explain.

Store your completed reflection in your Physics 20 course folder.

Module 7: Lesson 1 Assignment

Make sure you have completed all of the questions for the Lesson 1 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 7 assignments have been completed.

Lesson Summary

In this lesson you investigated the following questions:

- What is oscillatory motion?

- What is simple harmonic motion?

- What is the equation for the period of a weighted spring?

- How is simple harmonic motion related to circular motion?

Oscillatory motion is motion in which the period of each cycle is constant.

Simple harmonic motion is a case of oscillatory motion where the restoring force is proportional to the displacement of the mass relative to the equilibrium position. The displacement and force are always in opposite directions.

The period of a weighted spring is the time required to complete one cycle. It is defined by the spring constant and the mass of the weight attached to the spring. Mathematically, the relationship is expressed by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

The period for simple harmonic motion can be the same as that for circular motion. The radius of the circular motion is identical to the amplitude of simple harmonic motion.

Lesson Glossary

equilibrium position: the position where the resultant of all forces acting is zero

oscillate: to move back and forth at a constant rate

radian: a unit used to measure angles that is calculated as arc length divided by radius

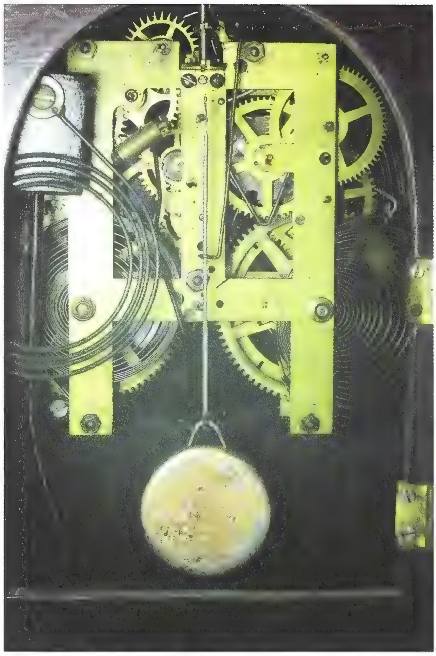
restoring force: a force that causes an object to return to an equilibrium position

simple harmonic motion: a repeating motion about a central equilibrium point caused by restoring forces

spring constant: a measure of the stiffness or strength of a spring (the *k* in Hooke's law)

Lesson 2—Simple Harmonic Motion of a Pendulum

Get Focused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You may not recognize it, but this photo shows an early version of the modern-day clock. It has a pendulum hanging in the centre. The pendulum, once disturbed, will oscillate back and forth, demonstrating simple harmonic motion with a constant period similar to that of a weighted spring. The oscillation of the pendulum controls the turning of the gears, and the gears turn the hands of clock.

In order to be useful, all clocks' hands have to move at a similar speed. Could you imagine if they didn't? It would be impossible to make appointments, show up for school, or follow any kind of a schedule. Nothing could be organized around time. Early clocks used a pendulum to control motion that was transmitted through a set of gears to move the hands of the clock at precise moments. Every clock had to do the same thing. How do you ensure this happens? Why is a pendulum a suitable method for keeping good time?

In this lesson you will investigate the following questions:
- Is the motion of a pendulum simple harmonic motion?
- What is the equation for the period of a pendulum?
- How is the restoring force of a pendulum determined?
- How is simple harmonic motion related to circular motion?

Module 7: Lesson 2 Assignments

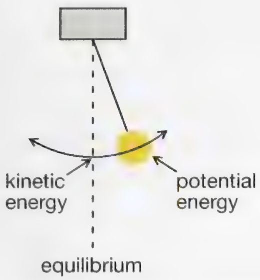
In this lesson you will complete the Lesson 2 Assignment in the Module 7 Assignment Booklet.
- Try This—TR 1, TR 2, TR 3, TR 4, TR 5, TR 6, and TR 7
- Lab—LAB 6, LAB 7, LAB 8, LAB 9, LAB 10, and LAB 11

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

Observing Simple Harmonic Motion



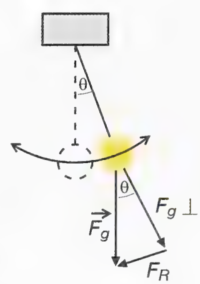
Hundreds of years ago, Galileo Galilei watched a candle-burning chandelier swinging back and forth on a long chain in a cathedral. Galileo wondered what determined the period of the chandelier. What effect would changing the length of the chain have? If the mass of the chandelier were changed, would that affect the period of vibration? If the amplitude (size of swing) was increased or decreased, would the period be affected? To answer these questions, Galileo did an experiment similar to the one you are about to do.

At rest, a pendulum hangs at the equilibrium position. The pendulum is given potential energy when the bob is pulled to one side. This would be considered the starting position. When it is released, the pendulum is pulled by gravity toward the equilibrium positiosn. The potential energy is converted into kinetic energy until the pendulum reaches its maximum speed (at the equilibrium position). The pendulum then continues through the equilibrium position to end up on the opposite side from where it started at rest for an instant. The kinetic energy has been converted back into potential energy. Then gravity pulls the pendulum back through the equilibrium position until it reaches its starting position again. One cycle or vibration has been completed.

A moving pendulum exhibits simple harmonic motion only when the amplitude of the motion is small. This means that when the amplitude is small, the pendulum is vibrating with a constant frequency or period of motion and there is a restoring force directed towards a central equilibrium point. Furthermore, for small displacements, the magnitude of the restoring force is proportional to the displacement from the equilibrium point (as described by Hooke's law).

Determining the Restoring Force of a Pendulum

When a mass is attached to the end of a hanging string and is pulled from its equilibrium position, its displacement is defined by angle θ (as illustrated here). When the pendulum is released, a component of the gravitational force will cause the mass to accelerate towards the equilibrium position, along the arc. This component is called the restoring force (F_R), and it can be expressed in terms of the gravitational force using the appropriate trigonometric function.



The restoring force of a pendulum is the component of gravity that acts along the arc to pull the mass back towards the equilibrium position. Expressed as an equation, it is

$$F_R = F_g(\sin\theta)$$
$$F_R = mg(\sin\theta)$$

Quantity	Symbol	SI Unit
restoring force	F_R	N
angle between pendulum and equilibrium point	θ	degrees
force due to gravity	F_g	N

This equation only works when a pendulum is exhibiting simple harmonic motion. When the angle of release is larger than 15°, the pendulum no longer exhibits simple harmonic motion. See page 361 of your textbook for a complete explanation of this fact.

Read

How is the simple harmonic motion of a pendulum different than the weighted spring studied in the previous lesson? Read "Simple Harmonic Motion of a Pendulum" on pages 359 to 362 of your textbook, and look for differences.

Example Problem 1

A 27.0-kg child plays on a swing, oscillating back and forth without pumping. What is the maximum restoring force that acts on the child when the swing oscillates with a maximum displacement of 10.0°?

Solution

The maximum restoring force occurs when the maximum displacement angle is reached. In this case, the maximum displacement is 10.0°.

$$F_R = mg(\sin\theta)$$
$$F_R = (27.0\text{ kg})(9.81\text{ m/s}^2)(\sin 10.0°)$$
$$F_R \doteq 45.99419282\text{ kg}\bullet\text{m/s}^2$$
$$F_R = 46.0\text{ N, correct to 3 significant digits}$$

Correct to 3 significant digits, the maximum restoring force that acts on the child is 46.0 N.

Module 7: Lesson 2 Assignment

Remember to submit the answers to TR 1, TR 2, and TR 3 to your teacher as part of your Lesson 2 Assignment in your Module 7 Assignment Booklet.

Try This

TR 1. Calculate the restoring force that acts on a 1.0-kg hanging mass when it is displaced 2.0° from equilibrium.

TR 2. Calculate the restoring force that acts on a 1.0-kg hanging mass when it is displaced 10° from equilibrium.

TR 3. Based on your answers from TR 1 and TR 2, what is the relationship between the magnitude of the restoring force and the angle of displacement from equilibrium?

Lesson 2 Lab: Equation for the Period of a Pendulum

Problem

What is the equation for the period of a pendulum?

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Simple Harmonic Motion Pendulum" in the search bar. Choose the item called "Simple Harmonic Motion: Pendulum Motion (Grade 11)." Open the simulation, and begin the Procedure.

You will use this simulation to complete the Period Measurements table. Save the table to your course folder. You will be updating the table in LAB 1, LAB 2, LAB 3, LAB 4, LAB 5, and LAB 6.

Number of Cycles	**Time to Complete (Seconds)**			
	With Default Settings	With Modified **Amplitude of Release**	With Modified **Mass**	With Modified **Spring Constant**
10				
1				

Procedure

Using the simulation, determine the period of the pendulum by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Press "Play," and carefully watch the pendulum move through ten complete cycles. (One cycle is complete every time the pendulum passes the starting position on its way up.)

LAB 1. Stop the simulation at the end of ten cycles. Use information from the data display to record the time for ten cycles in the Period Measurements table. Record your data under the column heading "With Default Settings."

The time required to complete one cycle is the period of the pendulum. Now you will systematically investigate the effects of changing the angle of release (A), the mass (kg), length (m), and acceleration due to gravity (g).

Using the simulation, determine if the period of the pendulum is affected by the angle of release by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Change the amplitude to the maximum value of 0.52 m ( ).
- Press "Play," and observe ten complete cycles. (One cycle is complete every time the pendulum passes the starting position on its way upward.)

LAB 2. From the data display, record the time for ten cycles in the Period Measurements table. Record the data under the column heading "With Modified Angle of Release."

On the simulation, determine if the period of the pendulum is affected by the mass by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Change the mass of the pendulum to the maximum value of 1.00 kg ( ).
- Press "Play," and observe ten complete cycles. (One cycle is complete every time the pendulum passes the starting position on its way upward.)

LAB 3. From the data display, record the time for ten cycles in the Period Measurements table. Record the data under the column heading "With Modified Mass."

On the simulation, determine if the period of the pendulum is affected by the length of the pendulum by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Change the length of the pendulum to the maximum value of 2.00 m ( ).
- Press "Play," and observe ten complete cycles.

LAB 4. From the data display, record the time for ten cycles in the Period Measurements table. Record the data under the column heading "With Modified Length."

On the simulation, determine if the period of the pendulum is affected by the acceleration due to gravity by doing the following:

- Reset the applet ( ).
- Turn on the data display ( ).
- Change the acceleration due to gravity to the maximum value of 20.0 m/s^2 ( ).
- Press "Play," and observe ten complete cycles.

LAB 5. From the data display, record the time for ten cycles in the Period Measurements table. Record the data under the column heading "With Modified Acceleration Due to Gravity."

Module 7: Lesson 2 Assignment

Remember to submit the answer to LAB 6 to your teacher as part of your Lesson 2 Assignment in your Module 7 Assignment Booklet.

Observations

LAB 6. Find the average time for the completion of one cycle for each of the previous steps of the procedure. (You do this by dividing the time for ten cycles by ten.) Place your results in the appropriate cells in the Period Measurements table.

Module 7: Lesson 2 Assignment

Remember to submit the answers to LAB 7, LAB 8, LAB 9, LAB 10, and LAB 11 to your teacher as part of your Lesson 2 Assignment in your Module 7 Assignment Booklet.

Analysis

LAB 7. Has the period changed as a result of changing the angle of release? Explain.

LAB 8. Has the period changed as a result of changing the mass? Explain.

LAB 9. Has the period changed as a result of changing the length of the pendulum? Explain.

LAB 10. Has the period changed as a result of changing the acceleration due to gravity? Explain.

LAB 11. Summarize your findings from LAB 7 to 10 by listing the parameters that *do* affect the period of the pendulum and the ones that *do not* have an effect.

Conclusion

The period of a pendulum is the time required to complete one cycle or swing. The equation is as follows:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Quantity	Symbol	SI Unit
period	T	s
length	l	m
acceleration due to gravity	g	m/s^2

For small angles, the period of a pendulum depends only on its length and the acceleration due to gravity.

Read

Where did the equation above come from, and why is the period of a pendulum useful? Read "The Period of a Pendulum" on pages 377 to 379 of the textbook to find out.

The pendulum had immediate application in several areas of science. In addition to keeping good time, simple pendulums were used to measure the strength of Earth's gravitational field at various locations on Earth's surface. Variations in the acceleration readings can be used to indicate the possible presence of heavy ores in the ground. In a sense, a precise pendulum is a metal detector!

Look over the next two examples to prepare for the Self-Check and Try This questions that follow.

Example Problem 2

If a pendulum is 80.0 cm long, what is its period and frequency of vibration?

Solution

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2\pi\sqrt{\frac{0.800\ \text{m}}{9.81\ \text{m/s}^2}}$$
$$T \doteq 1.794\ 280\ 586\ \text{s}$$
$$f = \frac{1}{T}$$
$$f = \frac{1}{1.794\ 280\ 586\ \text{s}}$$
$$f \doteq 0.557\ 326\ 433\ 6\ \text{Hz}$$

Correct to 3 significant digits, the pendulum has a period of 1.79 s and a frequency of vibration of 0.557 Hz.

Example Problem 3

If on a previously unexplored planet, a probe found that a 50.0-cm pendulum completed 20 swings in 33.6 s, what would be the acceleration of gravity on that planet?

Solution

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$g = \frac{4\pi^2 l}{T^2}$$
$$g = \frac{4\pi^2(0.50\ \text{m})}{\left(\frac{33.6\ \text{s}}{20}\right)^2}$$
$$g \doteq 6.993\ 767\ 291\ \text{m/s}^2$$

Correct to 3 significant digits, the acceleration of gravity on the planet is 6.99 m/s^2.

Self-Check

SC 1. Complete question 11 of “7.3 Check and Reflect” on page 380 of the textbook. Use 1.62 m/s^2 for the value of g on the Moon.

Check your work with the answer in the appendix.

Module 7: Lesson 2 Assignment

Remember to submit the answers to TR 4 and TR 5 to your teacher as part of your Lesson 2 Assignment in your Module 7 Assignment Booklet.

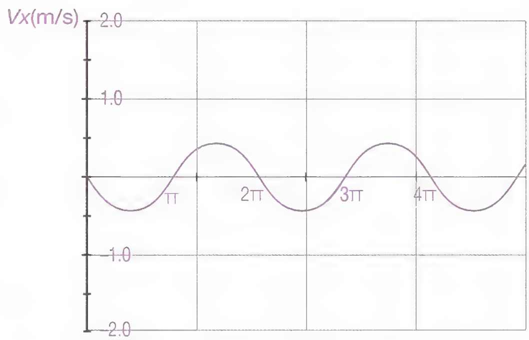
Try This

TR 4. Calculate the period of a 1.50-m pendulum. Verify your answer using the simulation.

TR 5. On the hypothetical planet Xeon, a pendulum with a length of 95.0 cm swings with a frequency of 1.50 Hz. What is the acceleration due to gravity on Xeon?

Simple Harmonic Motion Is Sinusoidal

You saw in Module 7: Lesson 1 that simple harmonic motion (SHM) is sinusoidal in nature.



SHM is also related to uniform circular motion (UCM). Did you notice that the *x*-axis in the simulation showed radian units? Why would this way of measuring angles be chosen? Could it be because the circumference of a circle is π times the diameter and the writers were trying to make the relationship between SHM and UCM more obvious? Or is it because it makes the formula easier to work with?

Look closely at the similarities between the SHM and the UCM equations. Manipulating the equation for the period of a pendulum in terms of acceleration gives $g = \frac{4\pi^2 l}{T^2}$, which is extremely similar to $a_{\text{inward}} = \frac{4\pi^2 r}{T^2}$ from uniform circular motion.

This is not surprising if you recall that UCM is vibratory, with a constant period and frequency. Since both equations describe periodic vibratory motion, they *should* have the same form.

Module 7: Lesson 2 Assignment

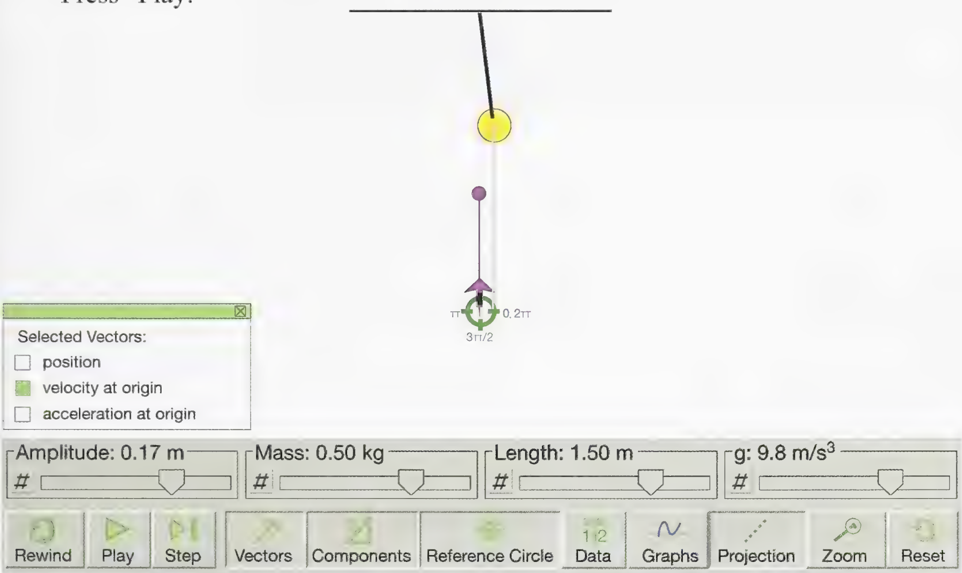
Remember to submit the answers to TR 6 and TR 7 to your teacher as part of your Lesson 2 Assignment in your Module 7 Assignment Booklet.

Try This

Go to **www.learnalberta.ca**. You may be required to input a username and password. Contact your teacher for this information. Enter the search terms "Simple Harmonic Motion Pendulum" in the search bar. Choose the item called "Simple Harmonic Motion: Pendulum Motion (Grade 11)." Reopen the simulation, if necessary, and complete the following questions.

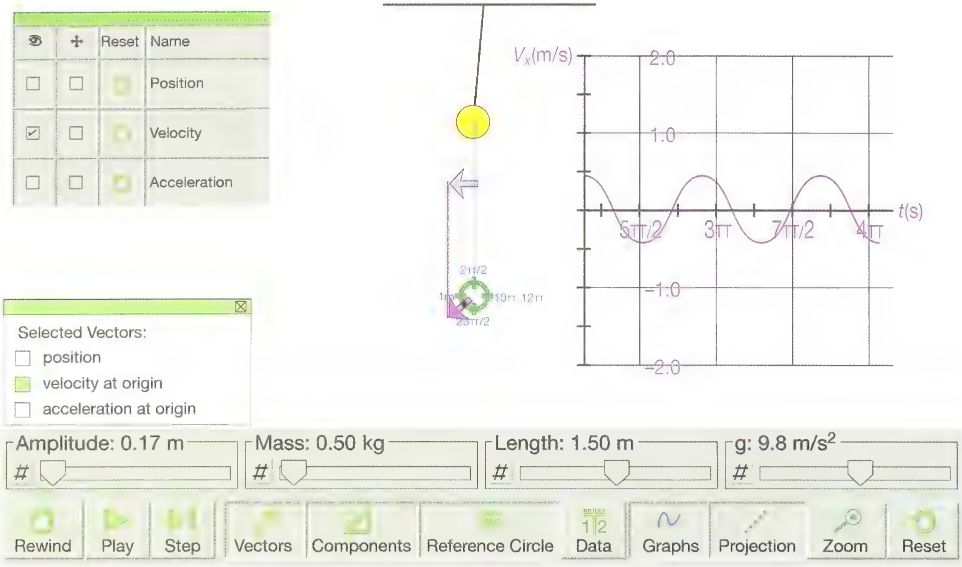
Observe uniform circular motion and simple harmonic motion at the same time by doing the following:

- Reset the simulation ( ).
- Display the "Reference Circle" ( ), "Vector Components" ( ), and "Projection" lines ( ).
- Display the "Vectors" menu ( ), and select "velocity at origin," as shown below. You may have to drag the green bar of the "Selected Vectors" popup toward the top of the window to see the choices.
- Press "Play."



TR 6. Describe one similarity and one difference between the velocity vector on the reference circle and the velocity vector on the pendulum.

Without changing any of the settings from TR 7, turn on the graphing function ( ), and select "Velocity" in the first box, as illustrated below. Drag the green bar on the top of the graphing popup to the left so you can see the graph and the pendulum clearly.

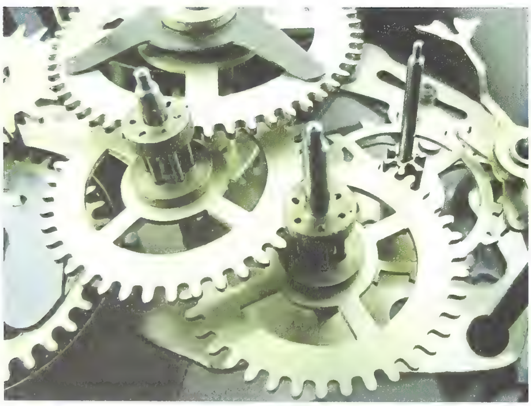


TR 7.

a. How many rotations does the reference circle make for every complete wave ( ) drawn on the graph?

b. How many complete swing cycles does this represent on the pendulum?

Reflect and Connect



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For a clock to be useful, it has to keep the same time as every other clock in the world.

If it runs too fast or too slow, it won't keep good time. The second hand must rotate once every 60 seconds, and the minute hand must complete one rotation in 60 minutes. The hour hand rotates once every 12 hours. How can a single pendulum, such as that seen in a grandfather clock, coordinate all of these hands moving at different rates? The simple harmonic motion of the pendulum is used to control a series of gears. The speed of each hand is controlled by the ratio of the gears. As long as the length of the pendulum and the acceleration due to gravity are constant, the pendulum will oscillate with a regular period.

This keeps the gears rotating at a regular speed and the hands of the clock moving just as you would want. Your clock gives you the regular motion needed by a timepiece.

At the bottom of the pendulum is a small adjustable weight. Moving this weight will change the period of the pendulum a small amount. If the clock is running fast, the adjustable weight is moved down on the pendulum. If the clock is running slow, the adjustable weight is moved up on the pendulum. (It's a quick way to change the pendulum's length.)

Discuss

A metronome is a timing device used to produce a periodic, audible tone or beat for establishing tempo in a musical performance. (Usually, a metronome is used when practising and not when performing.) The metronome to the right consists of a thin vertical bar that a small weight slides along (shown in the photograph near the bottom of the thin bar). Like the pendulum clock, if you move the weight closer to the end or the top of the pendulum, the pendulum will slow down. If you move the weight farther from the end of the pendulum by sliding the weight down to the bottom, the pendulum speeds up. You may think of this as an inverted pendulum, with a period that is much more easily adjustable than the pendulum clock. In the discussion forum, explain how moving the weight along the vertical bar causes the period to change. What variable in the equation for the period of a pendulum is being adjusted? How is the variable adjusted?

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Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will help you reinforce your learning about oscillation. Complete at least one of these reflection activities:

- Some early timekeeping devices are still used today. Others have been mostly forgotten. Use the Internet to research early timekeeping devices. Prepare an audio or video presentation about the devices you found that use regular, evenly spaced events.
- Create a short speech (1 to 2 minutes) about how music and timekeeping are related. Share it in the discussion area.

Store your completed reflection in your Physics 20 course folder.

Module 7: Lesson 2 Assignment

Make sure you have completed all of the questions for the Lesson 2 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 7 assignments have been completed.

Lesson Summary

As you worked through this lesson, you should have developed an understanding of simple harmonic motion and now be able to answer these questions:

- Is the motion of a pendulum simple harmonic motion?
- What is the equation for the period of a pendulum?
- How is the restoring force of a pendulum determined?
- How is simple harmonic motion related to circular motion?

A pendulum demonstrates simple harmonic motion when the amplitude of the motion is small and the restoring force is proportional to the displacement of the pendulum relative to the equilibrium position. The displacement and force are always in opposite directions.

The restoring force of a pendulum is the component of gravity that acts along the arc to pull the mass back towards the equilibrium position. Mathematically, the relationship is expressed by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The period of a pendulum is the time required to complete one cycle. It is dependent on the length of the pendulum and acceleration due to gravity. Mathematically, the relationship is expressed by

$$F_{\mathrm{R}} = F_{\mathrm{g}}(\sin\theta)$$
$$F_{\mathrm{R}} = mg(\sin\theta)$$

The period for simple harmonic motion can be the same as that for circular motion. The radius of the circular motion is identical to the amplitude of the simple harmonic motion. This fact is observed in the conversion of the simple harmonic motion of a pendulum into the uniform circular motion of the hands on a clock.

Lesson 3—Mechanical Resonance

Get Focused

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The modern wristwatch has come a long way in a relatively short time. In 1969, Seiko produced the world's first quartz wristwatch. Because it kept excellent time, something in the watch had to oscillate with a constant frequency. Recall that in a pendulum clock, the period of the pendulum was constant and it was used to control a set of gears that turned the hour and minute hands with constant frequency. In a similar way, the wristwatch needs an oscillator with a constant period. So, what do you put into a wristwatch that is tiny, oscillates with a constant frequency, and is unaffected by acceleration and movement? A precisely cut 3-mm quartz crystal "resonator" is perfect for the job! The next obvious question is, "Why?"

Chemically speaking, quartz is silicon dioxide. When it is cut in a specific shape and mounted correctly, it will bend in an electric field. When the electric field is removed, it will bend back, generating its own electric field and causing the motion to repeat. This relationship between deformation and electric fields is common in crystals and is called **piezoelectricity**. With the right shape and applied voltage, a quartz crystal will oscillate (bend back and forth) exactly 32 768 times per second for as long as the voltage is present. This constant, known frequency makes it ideal for driving the electric circuit of a modern wristwatch. This simple **resonant frequency** keeps everyone on the same time schedule, but it only works when the crystal is cut in a very precise shape. Why? What is a resonant frequency, and what is its relationship to the physical shape of an object? Do other, larger objects, such as bridges and buildings, oscillate with a precise frequency when disturbed?

piezoelectricity: the ability of certain crystals to change shape when a voltage is applied across them or to produce a voltage when a force is applied to them

resonant frequency: the frequency at which a system naturally vibrates

In this lesson you will investigate the following questions:

- What is a resonant frequency?
- What is mechanical resonance?
- Are there examples of mechanical resonance in your everyday world?

Module 7: Lesson 3 Assignments

In this lesson you will complete the Lesson 3 Assignment in the Module 7 Assignment Booklet.

- Try This—TR 1, TR 2, TR 3, TR 4, TR 5, TR 6, and TR 7
- Lab—LAB 1 and LAB 2

You must decide what to do with the questions that are not marked by the teacher.

Remember that these questions provide you with the practice and feedback that you need to successfully complete this course. You should respond to all the questions and place those answers in your course folder.

Explore

When you walk or run, your arms swing. You don't mean for this to happen, but it does. Without you thinking about it, your arms swing at a given frequency that feels natural to you and is based on your walking movement. This "natural" frequency is known as a *resonant frequency*. Any object or system that oscillates or vibrates does so at a unique frequency that depends on the physical characteristics of that object. For example, your arms probably don't swing at the same frequency as your friend's arms when you are walking side by side. (You could make them do this, but it will not feel natural.) And your arms will not just oscillate on their own; they have to be disturbed or caused to swing by your walking motion. A pendulum is an excellent example of this. (Your arm is, in fact, a sort of pendulum.) As you learned in the previous lesson, if a pendulum is disturbed, it swings back and forth with a constant period that is defined by the physical length of the pendulum. If you change the length, you change the period and the frequency at which the pendulum swings. Therefore, the physical size of the pendulum is related to its resonant frequency.


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However, without a force, your arms will not swing and a pendulum would hang motionless. A force is required to create the oscillations, and it has to be present to maintain the resonant frequency once the motion starts. The pendulum in a clock, for example, has to be "wound" occasionally. When this is done, a spring mechanism or a hanging weight in the clock is used to keep "pushing" the pendulum. This keeps it oscillating as friction acts to slow it down. This is similar to the "pumping" action you would exert on a swing set—as soon as you stop pumping your legs, the swing slows down and eventually stops. And you have to pump at precise moments on a swing—legs completely extended on the way up and feet tucked under on the way down. Any deviation from this pattern causes the swing to slow down and eventually stop. The pumping has to match the resonant frequency of the swing.

The pumping in this example is called a **forced frequency**. If this frequency is close to or matches the resonant frequency of the swing, the swing will oscillate with a large amplitude. This is known as **mechanical resonance**.

forced frequency: the frequency at which one object attempts to make a second object vibrate

mechanical resonance: the tendency of a system to oscillate at maximum amplitude at a specific frequency, known as the resonant frequency

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Resonance on Swing." See how a child on a swing illustrates mechanical resonance by watching the streaming video resonance on swing. Look for the answers in the video to the following two questions.

Self-Check

SC 1. A child on a swing is a kind of ____________.

Check your work with the answer in the appendix.

Module 7: Lesson 3 Assignment

Remember to submit the answer to TR 1 to your teacher as part of your Lesson 3 Assignment in your Module 7 Assignment Booklet.

Try This

TR 1. The periodic push that a child exerts on the swing must match the ___________ frequency of the swing.

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Resonance in Music." See how mechanical resonance is used in musical instruments by viewing the streaming video resonance in music. Look in the video for the answers to the following four questions.

Module 7: Lesson 3 Assignment

Remember to submit the answers to TR 2 and TR 3 to your teacher as part of your Lesson 3 Assignment in your Module 7 Assignment Booklet.

TR 2. *Resonance* stems from the Latin noun meaning ________________.

TR 3. Every oscillating system has a ___________ frequency, which is determined by the ______________ properties of the object.

Self-Check

SC 2. In the piano, the vibrating strings make the soundboard ____________.

SC 3. Small vibrations at the resonant frequency create a __________ amplitude vibration.

Check your work with the answer in the appendix.

Read

Read "Applications of Simple Harmonic Motion" on pages 381 to 383 of your textbook.

Lesson 3 Lab: Investigating Mechanical Resonance

To see mechanical resonance in action, complete "7-5 QuickLab: Investigating Mechanical Resonance" on page 384 of your textbook. If possible, obtain the materials from your teacher and perform the experiment with a classmate at school. If you are doing this lab at home, you can tie the string between two chairs and use unopened 200-g cans of food (e.g., flakes of tuna) in plastic bags for masses.

Module 7: Lesson 3 Assignment

Remember to submit the answers to LAB 1 and LAB 2 to your teacher as part of your Lesson 3 Assignment in your Module 7 Assignment Booklet.

LAB 1. Complete questions 1, 2, and 3 of "Part A" on page 384.

LAB 2. Complete questions 4, 5, and 6 of "Part B" on page 384.

Watch and Listen

Go to your Physics 20 Multimedia DVD, and watch the video called "Resonance in Buildings." See how mechanical resonance can destroy a building during an earthquake by viewing the streaming video resonance in buildings. Look for the answers in the video for the following two questions.

Module 7: Lesson 3 Assignment

Remember to submit the answers to TR 4 and TR 5 to your teacher as part of your Lesson 3 Assignment in your Module 7 Assignment Booklet.

Try This

TR 4. Compared to audible sound, the frequency of earthquake waves are generally ___________.

TR 5. Buildings between 5 and 40 storeys high are typically __________ with earthquake waves.

Read

Read more about how mechanical resonance is taken into account in construction on pages 385 and 386 in "Resonance Effects on Buildings and Bridges" of the textbook.

Module 7: Lesson 3 Assignment

Remember to submit the answer to TR 6 to your teacher as part of your Lesson 3 Assignment in your Module 7 Assignment Booklet.

TR 6. Engineers build energy ________ systems into buildings so that an earthquake will not destroy them easily.

Read

Read more about mechanical resonance in clocks on page 387 of your textbook.

Self-Check

SC 4. Complete question 13 of "7.4 Check and Reflect" on page 388 of your textbook.

Check your work with the answer in the appendix.

Module 7: Lesson 3 Assignment

Remember to submit the answer to TR 7 to your teacher as part of your Lesson 3 Assignment in your Module 7 Assignment Booklet.

TR 7. A 25.0-kg child in Red Deer pumps herself on a swing when she kicks upward on the downswing, thus changing the distance from the pivot point to her centre of gravity from 2.40 m to 2.28 m. What is the difference in the resonant frequency of her swing before the kick and afterwards?

Reflect and Connect

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The electrical force applied to a small, precisely shaped quartz crystal causes it to oscillate (bend back and forth). The electrical signal matches the resonant frequency of the crystal, causing it to oscillate with maximum amplitude. This mechanical resonance is used to generate an electrical signal that can drive the gears in an analog wristwatch or drive the electrical signals in a digital wristwatch. Whether analog or digital, the principles of mechanical resonance are used to keep accurate time. With many millions of timekeeping devices all over the world, it is easy to understand how your entire existence can be scheduled and coordinated with other organizations and people. In order for this to be effective and useful, the multitudes of timekeeping devices all over the planet need to be synchronized.

When the power to your house is interrupted during a storm, for example, all the clocks need to be reset to the current time. Where does this current time come from? Recall your work with global positioning satellites, which are based on accurate synchronization of time signals. Is there a "master clock" by which all other clocks on the planet are synchronized? What kind of clock is the most accurate on the planet? Do a quick Internet search for atomic clocks, and try to find out about the "master clock." Consider how important this clock is to our civilization and if resonance plays a role in its operation.

Discuss

The original Tacoma Narrows suspension bridge in Washington state was constructed in 1940. It lasted four months before self-destructing in dramatic fashion. Engineers had failed to consider mechanical resonance in the design. Go to your Physics 20 Multimedia DVD, and watch the video called "Resonance in Bridges." View the destruction of the bridge in the video resonance in bridges.

Was the Tacoma Narrows Bridge a very large musical instrument? In the discussion forum, explain why the bridge was destroyed and how scale models are used to test resonance effects in building and bridge structures.

Reflect on the Big Picture

Each of the Reflect on the Big Picture sections in this module will help you reinforce your learning about oscillation. Complete the concept map in "Conceptual Overview" on page 389 of the textbook. Add the concept map to your Physics 20 course folder.

Complete at least one of the following reflection activities:

- Brainstorm reasons why troops don't march across bridges but, rather, just walk. Look up your ideas on the Internet. Choose your top two ideas, and list them along with the URLs of websites that convinced you to rate them in your top two websites.
- You are playing some of your favourite music when it keeps getting interrupted with a buzzing somewhere in your listening room. What is happening? How can you explain it? Draw or paint a poster to show your situation, and explain the cause.
- Create a timeline of timepiece development. Include pictures and explanations of each development.

Store your completed reflection in your Physics 20 course folder.

Going Beyond

Just how sensitive to temperature is a common digital watch? Design and carry out an experiment to determine if changing temperatures affect the timing characteristics of the crystal in a digital watch. You will have to find a source of accurate time information and a way to vary the temperature of the watch over a long enough time to be sure of your results.

Module 7: Lesson 3 Assignment

Make sure you have completed all of the questions for the Lesson 3 Assignment. Check with your teacher about whether you should submit your assignment now or wait until all of the Module 7 assignments have been completed.

Lesson Summary

As you worked through this lesson, you were focusing on these questions:

- What is a resonant frequency?
- What is mechanical resonance?
- Are there examples of mechanical resonance in your everyday world?

Resonant frequency is the natural frequency of vibration for an object, defined by its physical characteristics, such as the length of a pendulum or the shape of a quartz crystal.

Mechanical resonance is the increase in amplitude of oscillation of a system as a result of a periodic force whose frequency is equal to or very close to the resonant frequency of the system.

Buildings, bridges, musical instruments, and clocks are all applications of mechanical resonance. The resonant frequency of such objects must be understood in order for them to work properly and safely.

Lesson Glossary

forced frequency: the frequency at which one object attempts to make a second object vibrate

mechanical resonance: the tendency of a system to oscillate at maximum amplitude at a specific frequency, known as the resonant frequency

piezoelectricity: the ability of certain crystals to change shape when a voltage is applied across them or to produce a voltage when a force is applied to them

resonant frequency: the frequency at which a system naturally vibrates

Module Summary

At the beginning of this module you were asked to think about the following question:

- What are examples of oscillatory motion in the world around you?

Oscillatory motion is motion in which the period of each cycle is constant.

Simple harmonic motion is a case of oscillatory motion, where the restoring force is proportional to the displacement of the mass relative to the equilibrium position. The displacement and force are always in opposite directions.

The period of a weighted spring is the time required to complete one cycle. It is defined by the spring constant and the mass of the weight attached to the spring. Mathematically, the relationship is expressed by

$$T = 2\pi\sqrt{\frac{m}{\mathrm{k}}}$$

The period for simple harmonic motion can be the same as that for circular motion. The radius of the circular motion is identical to the amplitude of the simple harmonic motion.

A pendulum demonstrates simple harmonic motion when the amplitude of the motion is small and the restoring force is proportional to the displacement of the pendulum relative to the equilibrium position. The displacement and force are always in opposite directions.

The restoring force of a pendulum is the component of gravity that acts along the arc to pull the mass back towards the equilibrium position. Mathematically, the relationship is expressed by

$$F_{\mathrm{R}} = F_{\mathrm{g}}(\sin\theta)$$
$$F_{\mathrm{R}} = mg(\sin\theta)$$

The period of a pendulum is the time required to complete one cycle. It is dependent on the length of the pendulum and acceleration due to gravity. Mathematically, the relationship for small angles is expressed by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The period for simple harmonic motion can be the same as that for circular motion. The radius of the circular motion is identical to the amplitude of the simple harmonic motion. This fact is observed in the conversion of the simple harmonic motion of a pendulum into the uniform circular motion of the hands on a clock.

Resonant frequency is the natural frequency of vibration for an object, defined by its physical characteristics, such as the length of a pendulum or the shape of a quartz crystal.

Mechanical resonance is the increase in amplitude of oscillation of a system as a result of a periodic force whose frequency is equal to or very close to the resonant frequency of the system.

Buildings, bridges, musical instruments, and clocks are all applications of mechanical resonance. The resonant frequency of such objects must be understood in order for the object to work properly and safely.

Module 7 Assessment

The assessment for Module 7 consists of three (3) assignments, as well as a module project:

- Module 7: Lesson 1 Assignment
- Module 7: Lesson 2 Assignment
- Module 7: Lesson 3 Assignment
- Module 7 Project

Module 7 Project

Choose one of the Reflect on the Big Picture activities that best represents your understanding of one of the concepts of this module. Reflect on your work now that you have completed the module, and rework where necessary. Provide reasons as to why and how you made this choice over one of the other Reflect on the Big Picture activities.

Module 7 Glossary

Glossary

equilibrium position: the position where the resultant of all forces acting is zero

forced frequency: the frequency at which one object attempts to make a second object vibrate

mechanical resonance: the tendency of a system to oscillate at maximum amplitude at a specific frequency, known as the resonant frequency

oscillate: to move back and forth at a constant rate

piezoelectricity: the ability of certain crystals to change shape when a voltage is applied across them or to produce a voltage when a force is applied to them

radian: a unit used to measure angles that is calculated as arc length divided by radius

resonant frequency: the frequency at which a system naturally vibrates

restoring force: a force that causes an object to return to an equilibrium position

simple harmonic motion: a repeating motion about a central equilibrium point caused by restoring forces

spring constant: a measure of the stiffness or strength of a spring (the k in Hooke's law)

Self-Check Answers

Lesson 1

SC 1.

Given

$f = 78$ Hz

Required

the period of the wings motion (T)

Analysis and Solution

$$\begin{aligned} f &= \frac{1}{T} \\ T &= \frac{1}{f} \\ &= \frac{1}{78\ \text{Hz}} \\ &= 0.013/\text{Hz} \\ &= 0.013\ \text{s} \end{aligned}$$

Paraphrase

The period of the hummingbird wing's motion is 0.013 s.

SC 2.

Given

$T = 0.00400$ s

Required

the frequency of the guitar string (f)

Analysis and Solution

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{0.00400\ \text{s}} \\ &= 250/\text{s} \\ &= 250\ \text{Hz} \end{aligned}$$

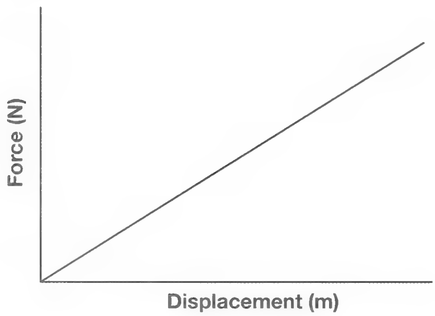
Paraphrase

The frequency of the guitar string is 250 Hz.

SC 3.

1. Controlled variables include the spring selected, the horizontal orientation, and the surface of the table. The manipulated variable is the displacement of the spring, and the responding variable is the force applied.

2. The following is a sample graph. Scales and slope will vary with the spring chosen.



3. The line of best fit is a linear relationship.

4. The slope should be taken from the line of best fit, not from individual data points.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{\Delta F}{\Delta x}$$

The units for the slope will be N/m.

5. The line should intercept the horizontal axis at 0 because there is no force applied, so the spring should not be stretched and the displacement should be zero.

SC 4. The spring constant is equal to the slope of the force-distance graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$\text{slope} = \frac{39\ \text{N} - 0\ \text{N}}{2.6\ \text{m} - 0\ \text{m}}$$
$$\text{slope} = 15\ \text{N/m}$$

The spring constant is 15 N/m.

SC 5.

Given

$x = 0.550$ m
$k = 48.0$ N/m

Required

the restoring force (F)

Analysis and Solution

The restoring force can be found from Hooke's law.

$$\begin{aligned} F &= -kx \\ &= -(48.0\ \text{N/m})(0.550\ \text{m}) \\ &= -26.4\ \text{N} \end{aligned}$$

Paraphrase

The restoring force of the spring is –26.4 N.

SC 6.

a. In the horizontal case, the restoring forces are only the tension and compression of the spring.

b. In the vertical case, the force of gravity (the weight of the object) must be taken into account.

SC 7.

Given

$m = 275.0$ kg
$x = 5.00$ cm

Required

the spring constant (k)

Analysis and Solution

Convert the displacement to metres. Divide the total mass by 4 to get the mass supported by each spring. Then use Hooke's law to find the spring constant.

$$
\begin{aligned}
-5.00\text{ cm} \times \frac{1\text{ m}}{100\text{ cm}} &= -0.0500\text{ m} \\
\frac{275\text{ kg}}{4} &= 68.75\text{ kg} \\
F &= -kx \\
k &= \frac{-F}{x} \\
k &= \frac{-mg}{x} \\
k &= \frac{-(68.75\text{ kg})(9.81\text{ m/s}^2)}{-0.0500\text{ m}} \\
k &= 1.35 \times 10^4\text{ N/m}
\end{aligned}
$$

Paraphrase

The spring constant of the car springs is 1.35×10^4 N/m.

SC 8. The velocity vectors are similar in their vertical magnitude. They are different in that the reference circle has both vertical and horizontal velocity while the weighted spring only exhibits vertical velocity.

SC 9. The acceleration and displacement are always pointing in opposite directions, so the force and displacement are also in opposite directions. The opposite directions are indicated mathematically using the negative in $-k\vec{x}$.

$$
\begin{aligned}
\vec{F} &= -k\vec{x} \\
m\vec{a} &= -k\vec{x}
\end{aligned}
$$

SC 10.

Given

$m = 0.724$ kg
$k = 8.21$ N/m
$\vec{a} = 4.11$ m/s^2 [left]

Required

the displacement of the mass at that acceleration (x)

Analysis and Solution

Choose the positive direction to be to the right. Then the acceleration will have a negative value. Use Hooke's law and Newton's second law to find the displacement.

$$\begin{aligned}
\vec{F} &= -k\vec{x} \\
m\vec{a} &= -k\vec{x} \\
x &= \frac{m\vec{a}}{-k} \\
x &= \frac{(0.724\,\text{kg})(-4.11\,\text{m/s}^2)}{-(8.21\,\text{N/m})} \\
x &= +0.362\,\text{m}
\end{aligned}$$

Paraphrase

The displacement of the mass is 0.362 m [right].

SC 11.

Given

$m = 50.0$ g
$k = 4.00$ N/m
$A = 1.12$ m

Required

the maximum speed of the mass

Analysis and Solution

Convert the mass to kilograms. The maximum speed can be found from the maximum kinetic energy. The maximum kinetic energy will equal the maximum potential energy, which occurs at the maximum displacement, the amplitude.

$$50.0\ \text{g} \times \frac{1\ \text{kg}}{1000\text{g}} = 0.0500\ \text{kg}$$

$$\begin{aligned} E_{\text{k}_{\text{max}}} &= E_{\text{p}_{\text{max}}} \\ \frac{1}{2}mv_{\text{max}}^2 &= \frac{1}{2}kx_{\text{max}}^2 \quad \text{but } x_{\text{max}} = A \\ \frac{1}{2}mv_{\text{max}}^2 &= \frac{1}{2}kA^2 \\ v_{\text{max}} &= A\sqrt{\frac{k}{m}} \\ v_{\text{max}} &= (1.12\ \text{m})\sqrt{\frac{4.00\ \text{N/m}}{0.0500\ \text{kg}}} \\ v_{\text{max}} &= 10.0\ \text{m/s} \end{aligned}$$

Paraphrase

The maximum speed of the mass is 10.0 m/s.

SC 12.

Given

$f = 3.50$ Hz
$k = 24.5$ N/m

Required

the mass of the object (m)

Analysis and Solution

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$m = \frac{T^2k}{4\pi^2} \quad \text{and } T = \frac{1}{f}$$

$$m = \frac{k}{4f^2\pi^2}$$

$$m = \frac{24.5\ \text{N/m}}{4(3.50\ \text{Hz})^2\pi^2}$$

$$m = 5.07 \times 10^{-2}\ \text{kg}$$

Paraphrase

The object has a mass of 5.07×10^{-2} kg.

Lesson 2

SC 1.

Given

$l = 50.0$ cm
$g = 1.62\ \text{m/s}^2$

Required

the period of the pendulum (T)

Analysis and Solution

The 50.0 cm length is 0.500 m.

$$T = 2\pi\sqrt{\frac{l}{g}}$$
$$T = 2\pi\sqrt{\frac{0.500\ \text{m}}{1.62\ \text{m/s}^2}}$$
$$T = 3.49\ \text{s}$$

Paraphrase

The period of the pendulum on the Moon is 3.49 s.

Lesson 3

SC 1. A child on a swing is a kind of **pendulum**.

SC 2. In the piano, the vibrating strings make the soundboard **resonate**.

SC 3. Small vibrations at the resonant frequency create a **large** amplitude vibration.

SC 4.

a. Two advantages of a quartz clock are that it is more accurate than a pendulum clock and it never needs to be wound.

b. A disadvantage of a quartz clock is that it cannot work without a battery.